

## **Evaluation of Homogeneous Clusters within a Probabilistic Composition of Preferences**

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### **Abstract**

When employing multiple criteria to rank a set of options in a unique ordering, the possibility of correlation between the criteria must be taken into account. This correlation is unavoidable in the case of two criteria that consider the same attribute, with the only difference that one of them measures such attribute in individuals isolated and others measure them in clusters of the same set of individuals. This last situation is engendered by the use of the evaluation to enhance collective efforts of individuals otherwise worried only with their separate positions in the ranking. Here an overview of the use of a probabilistic approach to deal with the composition of criteria facing such difficulties is made. An example of application is presented.

**Keywords:** probabilistic model – fuzzy logic – preferences – multicriteria decision analysis – DEA – probabilistic composition

### **1. Introduction**

Several difficulties have always been recognized in the combination of multiple criteria. The inconvertibility of evaluations on different measurement scales, the lack of precision in the observed values, the dependence between the criteria generate the most important of such difficulties. Notwithstanding, the combination of criteria to rank options with distinct features is always larger. Performance monitoring by numerical indices is necessary to determine differentiated resources allocations, policy goals and so on.

The importance of the inconvertibility of evaluations increases as these evaluations extend from the economic sector, where the convertibility effort is most of the time limited to set all values in monetary terms, to other areas where social, environmental and other values are not suitable to conversion to monetary representation.

The difficulty to deal with criteria evaluated in inconvertible reference settings is clear when the combination is made by the traditional form of weighted averages. The need to take into account the scale of measurement when setting the weights has been signaled in the literature since long ago. Foster and Sen (1997) and Woodward and Bishop (1997) contain insightful views and references to previous works on that. More recent reviews may be found in Saisana et alii (2005) and Zhou et alii (2006), for instance.

The imprecision of the evaluations and the need to take into account uncertainty and derive probabilistic conclusions are also important points of criticism to combined criteria evaluations. See, for instance, Banker (1993) and Selvanathan and Prasada Rao (1994). In fact there is always some amount of subjectivity in deriving preference for any kind of attribute and the lack of information on the degree of uncertainty in uncertain classifications considerably impairs the use of such classifications.

The transformation into probabilities of being ranked first, proposed in Sant'Anna and Sant'Anna (2001) opens a way to overcome such difficulties. It starts by ranking according to the different particular criteria to be combined. The imprecision in such rankings is modeled by considering the ranks as midpoints of statistical distributions determined by adding stochastic disturbances to them. Finally probabilities of attaining the first position are computed. These probabilities can be combined into global measures without the need of assigning weights to the criteria.

Another important source of criticism to combined evaluations is the presence of unaccounted dependence between the variables combined in global indices. The application dealt with in this paper is particularly suited to the presence of such kind of dependence. If the same attributes are measured in the analysis applied to evaluate the performance of individuals isolated and their performance as a group, the same disturbances must be present in the formation of the two evaluations. In the probabilistic approach, the correlation between the criteria may be directly taken into account if the global measurement is given in the form of joint probabilities.

This paper studies the application of the probabilistic composition to the situation where the individuals evaluated interact inside groups. The individual performances must be evaluated taking into account group features. An example of application to data on clients of a supermarkets chain is presented.

The paper is developed as follows. In the next Section, the transformation into probabilities of being the first is described. Section 3 presents the different points of view that may be taken in combining such probabilities in global evaluations. Section 4 treats the relations of the probabilistic approach with Data Envelopment Analysis and Section 5 the problem of correlation between the partial evaluations. Section 6 presents the problem of taking into account collective evaluations and Section 7 shows an example of application of the probabilistic approach to this problem. Final comments conclude the paper.

## **2. Probabilities of being the first option**

The key computation in the evaluation of probabilistic preferences is the transformation into probabilities of being the first in a sample. The probability of choosing a particular option as the best one is a natural measure of the decision maker preference for that option. Nevertheless, we frequently start from other measurements. The simplest starting point is the ordering of the options. For the measurement of preferences based on the level or degree of presence of some attribute, the relative position of the options may be derived from numerical values of costs or distances, for instance. In other situations there is no such quantifiable attribute and the preferences are given in terms of common language, such as low, moderate or high preference.

The imprecision in this last case, of qualitative evaluations, is usually taken into account by means of the representation through fuzzy intervals (Zadeh, 1965), but it is also present in ordinal and cardinal scales and can be represented analogously. To compute the probabilities of being the first option all we need is, besides an ordering (with ties admitted as well as different distances between successively ordered options), a statistical measure of the uncertainty on each position in that ordering. The modeling of the uncertainty may be always done in the measurement with error framework. The rank of the option (or any other

numerical indication of its position) is thought as a centrality parameter of a statistical distribution. The observed range may be used as an estimate for a common range for the distributions relative to each individual measurement. Different assumptions on the form may be taken to complete the modeling of these probability distributions.

To make easier the comparisons, the probabilities of being the first may be computed with respect to a sample of fixed size, randomly generated or withdraw in fixed percentiles of the set of values attributed to the options under evaluation. For instance, this sample may be formed by the nine deciles of this distribution. This has the advantage of presenting evaluations always distributed around the value 0.1 that will be given to all options if they are indiscernible.

In the case of a Likert scale of five points, representing the five possible evaluations by the numbers 1 to 5, the intermediate deciles will be 1.5, 2.5, 3.5, 4.5. The distribution centered in each of these values may be a triangular distribution with extreme values 0.5 and 5.5. Or a normal distribution with standard deviation derived from the observed range, as used in Sant'Anna (2005). Or a uniform distribution with a range determined in such a way as to allow for all inversions of ranks considered reasonable, as in Sant'Anna (2002).

The probabilities of being the first option can be computed by integrating with respect to the joint density the probability of the option under evaluation presenting a value better than that of each other option. To compute this probability we ought to divide the range into intervals bounded by the values in the sample.

Let us consider, for instance, the case of triangular distributions, centered at the observed values and with extremes fixed at 1/8 of the absolute distance between the first and the ninth decile of the observed distribution, and let us assume independence between the disturbances affecting the evaluations of different options according to the same criterion. Then, the probability of being the highest, for an option ranked  $i$ -th (in increasing order) in the observed sample of evaluations according to the criterion  $X$  will be obtained by adding integrals of terms of the form  $\Pi[1-(1-x)^2/(1-a_p)] \Pi(x^2/a_q)$  where the first product is for  $p < j$  and the second for  $q > j$ ,  $p$  and  $q$  different from  $i$ , and for  $j$  varying from 0 to  $n$ , the number of observations in the sample. The integration will be with respect to the density of  $X(i)$ . This density is equal to  $2(1-x)/(1-x_{(i)})$  for  $i < j$  and to  $2x/x_{(i)}$  for  $i > j$ .

An additional advantage, besides the advantages inherent in taking into account uncertainty, is derived from the transformation from ranks to probabilities of being the best or the worst option. This transformation increases distances between the most important options. Barzilai et alii (1987), Brugha (2000), Lootsma (1998), Tryantaphilou et alii (1994), among others, present good reasons to prefer nonlinear scales with these characteristics.

### 3. Combination of Probabilistic Preferences

A way to derive from the probabilities of being the first associated to each criterion a unique measure of global preference consists of treating these probabilities as conditional on the choice of the respective criterion and compute the total probability preference by adding the products of these conditional probabilities by the probabilities of choice of each criterion. The difficulty in this approach is to determine the marginal probabilities of choice of each criterion. This is specially difficult if the criteria are correlated. If it is possible to

rank the criteria and model the correlation between them, these probabilities of choice of each criterion may be computed in the same way the probabilities of preference according to each criterion are computed.

Dependence between the criteria may be directly taken into account if the global preference is determined in terms of joint probabilities of preference according to the multiple criteria. Different joint probabilities may be employed, depending on the point of view adopted. The different points of view may be characterized in terms of choice between extreme positions in two basic orientation axes. These extreme positions are, in one axis, an optimistic versus a pessimistic position and, in the other, a progressive versus a conservative position.

In the progressive-conservative axis, the evaluator pays attention to the probabilities of maximizing preference. The progressive evaluator looks after options that are the first in excellence, the conservative evaluator evaluates them by their ability of not minimizing the preference. The term 'conservative' in this terminology is related to the idea of avoiding losses, while the term 'progressive' is related to the idea of improving, of reaching higher patterns.

In the optimistic-pessimistic axis, the optimistic extreme consists of considering enough the satisfaction of only one criterion. All the criteria are taken into account, but the composition employs the connective 'or'. The joint probability computed is that of maximizing (in a progressive composition, or of not minimizing in a conservative one) the preference according to at least one of the multiple criteria. On the opposite end, the pessimistic preference goes for options that satisfy every criterion. The connective is 'and'. The joint probability computed is that of maximizing (or not minimizing) simultaneously the preference according to all the criteria. The terms optimistic and pessimistic are related to the idea of confiding that the most favorable or the less favorable criterion, respectively, will prevail.

By combining the positions in the extremes of these two axes, four different measures are generated. If the criteria are divided into groups and different points of view are allowed in the computation of the joint probabilities within each group, the number of possibilities increases. A natural division of the criteria into groups is in criteria for which the optimum is large and criteria for which optimization means reduction. For instance, criteria of benefits and criteria of disadvantages, criteria related to the production of outputs and criteria related to the use of inputs, criteria related to outcomes and criteria related to costs, and so on.

#### **4. Probabilistic Composition and Data Envelopment Analysis (DEA)**

The probabilistic approach here applied has in common with DEA the feature of deriving the evaluations from distances to the frontier. The computation of the probabilities of being the first is more robust because it involves comparison with all options, not only those in the frontier. From the points of view that may be chosen to combine the probabilistic evaluations, the point of view optimistic and progressive is closer to the DEA point of view. If this is the point of view chosen, DEA algorithms may also be employed to combine the partial probabilistic evaluations in a final aggregate value.

Generally, the use of DEA in multiple criteria composition follows the tradition of DEA by first identifying inputs and outputs and then constructing an aggregated index using the

common DEA procedure. Recent examples of such studies include Drake et alii (2006), Ramanathan (2006), Zaim, 2004, Zhou et alii (2007). This corresponds, in the probabilistic composition point of views, to divide the criteria into two blocks, one referring to the frontier of large values and the other to that of small values.

But the scope of DEA has broadened considerably over the last two decades. With all the criteria in the same direction, as benefit or cost variables, and then aggregated by a DEA constant inputs or constant outputs model, as developed by Caporaletti et alii (1999) or Lovell and pastor (1999), Cherchye et alii (2004) provides a list of other applications. This tendency may be due to the great advantage of DEA of not asking for weights for the criteria. Nevertheless, DEA derives weights that are different for each option under evaluation which depend on the part of the frontier to which the option is closer. By comparing to different ideal reference options, the ranking derived from DEA is questionable, mainly if there are different scales, importance or variability in the criteria.

Besides, DEA optimistic foundation of allowing variables weights, the evaluation of each option applying the weights more favorable to that option may lead to not taking into account some criteria. In the case of simultaneous evaluation of individual performances and cluster's performances, that will result in the individuals with performance above the average being evaluated by their individual performances while those with performances below the average are evaluated by the aggregate attributes. In the DEA framework an exit to avoid that is given by constraining the weights on each individual criterion to stay below that given to the same criteria when applied to the clusters. In the probabilistic approach a more precise treatment to this problem may be given by taking into account the correlation between the criteria.

Another important criticism to DEA is driven to the lack of statistical evaluations. Different efforts have been directed to associate confidence intervals and test hypotheses on the efficiency measurements. Basic issues on this subject are raised in Banker (1993) and Simar and Wilson (1998). With respect to that, an advantage of the transformation into probabilities of being the first is that it takes into account from the beginning the uncertainty in the measurements.

## **5. Dependence between Criteria**

In Sant'Anna (2008), it was verified that the composition of fuzzy logic (Zadeh, 1978) by the necessity and possibility concepts, that is equivalent to taking, respectively, the minimum and the maximum of the pertinence probabilities, corresponds to an extreme of the correlation between indicators of occurrence. In that extreme of maximal correlation that leads to the composition by the minimum, the composition by the joint probability will result in a ranking corresponding to the DEA approach of allowing each option to be evaluated by the most favorable criterion. The other extreme corresponds to the assumption of independence between the criteria.

The ranks derived from these two extreme assumptions constitute information that may be used complementarily. Moreover, correlation structures in an intermediary position between those two may also be explored. For instance, a composition approach may be established to employ a subjective contribution of experts only to rank the criteria. Based on this ranking, a small number of successive correlations may be estimated.

Independence between criteria applied to individuals isolated may be assumed and, after computing the joint probability of preference according to these criteria, the criteria related to collective evaluations may enter successively in the computation. The small number of correlations needed in this second stage may be estimated.

## **6. Modeling Cooperative Attributes**

Evaluation systems based on the comparison of individual performances may fail to attend the main objective, of enhancing global improvement, by fostering competitive practices where cooperation would be a more important asset. On the other end, leaving unrecognized individual efforts and evaluating only on the basis of large groups achievements may leave out of the performance evaluation important drives for improvement.

For instance, stimulating the productivity in scientific research by offering grants only to researchers presenting, comparatively, the best results on a list of indicators stimulates two kinds of attitudes that will harm the development of productive research activity. The first is the detachment of the individuals' research from the objectives of their institutions, which should be the real core of the most important research projects. The second is developing an opposition of each researcher to the success of the pairs which compete for the grants reserved for a same research field.

The evaluation system, even when designed to assign resources to individuals, must take into account variables measuring environmental variables that affect collectively groups of individuals or are affected by the joint action of such groups. By not taking into account social features affecting the performances evaluated, the evaluation will be unfair not only to the groups as a whole but even to the individuals compared.

By not taking into account the environmental conditions affecting the activities in the community where they are located, the evaluator that claims to be judging individual productivity may be only measuring individual results attributable to the context where the work is done and not to personal contributions. Sometimes the absolute results are obtained without any productivity of the individual in efficiently exploring resources made available by other sources and on which distribution neither the evaluator nor the evaluated person have any interference.

Here is developed a form to join, in the same evaluation system, individual and group indicators, in such a way that the evaluation of each individual is affected by the group performance but individual contributions have a significant impact on their particular evaluation. This system puts together variables measuring individual attributes with variables measuring the same attributes in aggregate units of evaluation. Thus, positive correlation between the stochastic components of these variables will probably be present.

The key feature of this system is then handling the correlation between criteria applied to clusters of options and criteria applied to individual options. A first principle in modelling the correlation in this context will be assuming maximal dependence between cluster indicators and the respective individual indicators. Even if not measuring the same feature, cluster evaluations being more affected by environmental stochastic factors must be more correlated among themselves and with the individual evaluations than these among themselves. A final important aspect to take into account is the advantage of assuming

independence to let numerical differences possibly registered being fully taken into account.

Another aspect that may be explored to make the evaluation adequately take into account the context, though centring attention in individual characteristics, is ranking in terms of evolution of the individual position through time. Färe et alii (1997) have shown that the Malmquist approach of evaluating evolution through time by computing indices to each option relatively to values of the other options fixed on successive time points may be employed in the DEA context. It may be employed by the same way in the probabilistic composition.

## **7. An Example of Application**

In this section a model for combining in an evaluation system individual and cluster criteria is developed. Three criteria are employed to evaluate clusters formed according to two different classification rules. The example is built in the context of evaluating clients of a retail sales chain. The objective is to enable the firm provide customized treatment to different classes of costumers. The same framework can however be employed to model evaluation in many other contexts.

The first aggregate variable,  $C_1$ , is given by a classification on 5 a priori levels, each with the same number of costumers, determined from the observed value of the transactions of the client with the network. Once the system is applied in a given time moment, this first variable may be the classification provided by the model in the last application of the system. In a context where the objective is to reduce inequality the preference in terms of this variable may be stated in an inverse order.

This volume criterion is complemented by a classification in terms of diversity,  $C_2$ . This second kind of classification is formed, in the case of network costumers, by counting the number of stores visited by the costumer in the last year. In a context of productivity evaluation this second variable may be thought as representing areas of actuation. Then those areas where smaller values for the individual indicators are expected would receive higher preference values.

The third variable,  $C_3$ , is designed to determine an intermediary level of aggregation. The clusters are formed by the intersection of the clusters determined by the two preceding variables. Costumers are ranked inside the clusters determined by the second variable according to their value in the first, or equivalently in the reverse order.

The individual evaluation variables are derived from the Recency, Frequency and Monetary value (RFM) approach to access importance of costumers to firm (Hughes, 2005). The first,  $C_4$ , is a recency variable, classifies costumers in decreasing order according to the number of days from the date of the last transaction of the year to the end of the year. The probabilistic transformation is to the probability of minimizing such number. The second,  $C_5$ , is a frequency variable given by the number of visits to the chain during the whole year. And the third,  $C_6$ , is a diversity variable given by the number of products of distinct classification bought by the client during the year.

Independence between the measurements representing the individually accessed variables can be accepted without questioning because such measurements involve observing behaviour at distinct circumstances. Since, in the present case, the individual variables are

not directly aggregated into any of the aggregate variables it is conceivable that stochastic independence may hold between the two kinds of variables. Even between the variables measured in an aggregate level, it may be assumed that errors in the measurement of the two first are independent. Thus, modelling the dependence structure between the criteria in this model is a considerably open question.

Table 1 shows, for two successive years, the correlations between the initial classification in five strata employed in the first criterion and a final classification in five strata of equal size derived from the final ranking derived from four hypotheses on dependence suggested by the reasoning above developed. The hypotheses confronted are ( $\wedge$  in the labels denoting composition by the minimum of the probabilities and  $*$  denoting composition by the product):

- Independence only between individual evaluations ( $C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5 \wedge C_6$ )
- Dependence only between the two sets of variables ( $C_1 \cdot C_2 \cdot C_3 \wedge C_4 \cdot C_5 \cdot C_6$ )
- Dependence only between aggregate evaluations ( $C_1 \wedge C_2 \wedge C_3 \cdot C_4 \cdot C_5 \cdot C_6$ )
- Independence between all evaluations ( $C_1 \cdot C_2 \cdot C_3 \cdot C_4 \cdot C_5 \cdot C_6$ )

Year	$C_1 \wedge C_2 \wedge C_3 \wedge C_4 \cdot C_5 \cdot C_6$	$C_1 \cdot C_2 \cdot C_3 \wedge C_4 \cdot C_5 \cdot C_6$	$C_1 \wedge C_2 \wedge C_3 \cdot C_4 \cdot C_5 \cdot C_6$	$C_1 \cdot C_2 \cdot C_3 \cdot C_4 \cdot C_5 \cdot C_6$
1	0.66	0.89	0.94	0.91
2	0.81	0.89	0.86	0.90

Table 1 – Correlation between Final and Initial Classifications

Table 1 reveals that the difference between the results derived from different assumptions is small. But it is clear that the hypothesis of independence only between the individually evaluated criteria, that means, dependence between the criteria evaluating clusters and dependence between the evaluation according to these criteria and that according to the criteria evaluating the individuals isolated, is the hypothesis that leads to a classification less correlated to the initial classification. So, to allow for the maximal refreshment of positions, this is the hypothesis to be assumed.

## 8. Final Comments

The modeling approach above developed shows the suitability of the probabilistic composition of preferences to combine criteria applied on different levels of aggregation. The application made can be extended to a large number of variables without any conceptual change.

The example studied brings a basic framework for the exploration of the dependence relations between criteria. Only extreme dependence relations were assumed. Efforts should be taken to obtain a quantitative basis of information on possible intermediary correlation structures.

The application to other instances of the same problem should bring new opportunities of development. Important areas of possible application are in the public sector, where evaluation must frequently face the need to take into account criteria unrelated to simple



quantitative attributes. An important feature of the evaluation system here developed is its full independence of the availability of numerical measurements to start with.

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